Pergamon

Int. J. Heat Mass Transfer. Vol. 39, No. 10, pp. 2191–2193, 1996

0017-9310/96 $15.00 + 0.00

0017-9310(95)00284-7

Approximate method for combined forced-convection and radiation heat transfer in absorbing and emitting gases flowing in a black, plane-parallel duct

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(*Received* 29 *September* 1994 *and in final form* 6 *April* 1995)

1. INTRODUCTION

During the past several decades, a considerable amount of theoretical studies have been made on combined forced-convection and radiation heat transfer in radiating gas tube flow with relation to thermal designs of rocket nozzles, high-temperature gas-cooled reactors and so forth[1–3]. It is, however, well recognized that theoretical results obtained for the heat transfer characteristics of such a physical system cannot be represented by simple analytical correlations involving relevant system parameters, because of nonlinearity of radiative heat transfer with respect to temperature and, thus, whenever a piece of information about the heat transfer characteristics is needed for system parameters which have not been examined in the literature, new analyses are always required.

For this reason, development of approximate methods for estimating the composite heat transfer characteristics are of great practical importance. The purpose of the present note is to fill this need.

To this end, first we describe an exact formulation, on the basis of the boundary layer theory, for analyzing combined forced-convection and radiation heat transfer in radiating gases flowing in a black, plane-parallel duct, and then propose an approximate one-dimensional method for predicting the heat transfer characteristics of this system. Moreover, with the proposed approximate method, the mixing cup temperatures and total Nusselt numbers are calculated for several typical system parameters and the obtained results are compared with the exact ones.

2. TWO-DIMENSIONAL FORMULATION BASED ON THE BOUNDARY LAYER THEORY

The physical system considered in the present study is based on the following assumptions:

(1) The system is bounded by black, plane-parallel plates of infinite length and infinite width. Both plates are maintained at the same constant temperature T_w.

(2) The flowing medium is a gray radiating gas.

(3) The fluid is incompressible and the physical properties of the medium are constant.

(4) The flow field is fully-developed laminar from the inlet of the duct.

(5) The temperature of the medium at the inlet of the duct is uniform and equal to T_0.

(6) The boundary layer approximation holds to be true: the Peclet number defined by $2\rho c_p u_m y_0/k$ is much greater than unity.

(7) The radiative heat transfer can be treated as if it were one-dimensional, i.e. $\partial q_{ry}/\partial_y \gg \partial q_{rx}/\partial x$. This assumption is justified, provided that the radiation Peclet number defined by $2u_m y_0/(16\sigma T_w^3/3\kappa\rho c_p)$ is appreciably greater than unity [4].

Under these assumptions, the two-dimensional energy equation is described as

$$\rho c_p u \frac{\partial T}{\partial x} = k\frac{\partial^2 T}{\partial y^2} - \operatorname{div}\mathbf{q}_r, \tag{1}$$

where u represents the velocity profile given by

$$u = 6u_m\{(y/y_0)-(y/y_0)^2\}. \tag{2}$$

The divergence of the radiative heat flux vector is written in the form

$$\begin{aligned}\operatorname{div}\vec{\mathbf{q}}_r &= \partial q_{ry}/\partial y \\ &= \kappa\{4\sigma T^4 - 2[\sigma T_w^4(E_2(\tau)+E_2(\tau_0-\tau)) \\ &\quad + \kappa\int_0^{y_0}\sigma T^4(y')E_1(|\tau-\tau'|)\,dy']\}.\end{aligned} \tag{3}$$

The boundary conditions for equation (1) are

$$x = 0: T = T_0 \quad y = y_0: T = T_w. \tag{4}$$

To obtain the dimensionless energy equation, we introduce the following dimensionless quantities:

$$\left.\begin{aligned} &N_R = k/4\sigma T_w^3 y_0 \quad Pr = \mu c_p/k \quad Re = 2y_0 u_m/\nu \\ &U = u/u_m \quad \eta = y/y_0 \quad \theta = T/T_w \quad \theta_0 = T_0/T_w \\ &\xi = (x/2y_0)/RePr \quad \tau = \kappa y \quad \tau_0 = \kappa y_0. \end{aligned}\right\} \tag{5}$$

With these quantities, equation (1) is rewritten as

$$\begin{aligned}\tfrac{1}{4}U\frac{\partial\theta}{\partial\xi} = \frac{\partial^2\theta}{\partial\eta^2} - \frac{\tau_0}{2N_R}\{&(\theta^4(\eta)-1)[E_2(\tau)+E_2(\tau_0-\tau)] \\ &+\tau_0\int_0^1[\theta^4(\eta)-\theta^4(\eta')]E_1(|\tau-\tau'|)\,d\eta'\}.\end{aligned} \tag{6}$$

NOMENCLATURE

a_0	expansion coefficient
c_p	specific heat of fluid
$E_n(x)$	exponential integral function of nth order
h_x	local total heat transfer coefficient
k	thermal conductivity of fluid
n	exponent
N_R	conduction–radiation parameter
Nuc	convective Nusselt number
Nur	radiative Nusselt number
Nut	total Nusselt number
Pr	Prandtl number
q_{cw}	convective heat flux at the wall
$\mathbf{q}_r$	radiative heat flux vector
q_{rw}	radiative heat flux at the wall ($y = 0$)
q_{rx}	x-component of the radiative heat flux vector
q_{ry}	y-component of the radiative heat flux vector
Re	Reynolds number
T	temperature
u	velocity
U	dimensionless velocity
x	x-coordinate
y	y-coordinate
y_0	duct spacing.

Greek symbols

η	dimensionless y-coordinate
θ	dimensionless temperature
κ	absorption coefficient
μ	viscosity
ν	kinematic viscosity
ξ	dimensionless x-coordinate
ρ	density of fluid
σ	Stefan–Boltzmann constant
τ	optical distance
τ_0	optical thickness
Ψ_{rw}	dimensionless radiative heat flux at the wall ($\eta = 0$).

Subscripts

m	mean value
w	wall value
0	inlet value
ξ	local quantity.

Similarly, the boundary conditions is rewritten in the form

$$\xi = 0 : \theta = \theta_0 \quad \eta = 0,1 : \theta = 1. \tag{7}$$

Once the temperature fields in each ξ-location are obtained by an appropriate method, the mixing cup temperatures and heat transfer characteristics can be readily estimated from the temperature profiles. The mixing cup temperature T_m is evaluated from

$$T_m = \int_0^{y_0} uT\,dy/y_0 u_m \tag{8}$$

and is rewritten as

$$\theta_m = T_m/T_w = \int_0^1 U\theta\,d\eta. \tag{9}$$

The convective and radiative heat fluxes at the wall ($y = 0$) are, respectively, written as

$$q_{cw} = -k\left.\frac{\partial T}{\partial y}\right|_{y=0} \tag{10}$$

$$q_{rw} = \sigma T_w^4[1-2E_3(\tau_0)] - 2\tau_0\int_0^1 \sigma T^4(\eta')E_2(\tau_0\eta')\,d\eta'. \tag{11}$$

The local total heat transfer coefficient is defined as

$$h_x = (q_{cw}+q_{rw})/(T_w - T_m). \tag{12}$$

With this quantity, the local total Nusselt number can be defined as

$$Nut_\xi = h_x 2y_0/k = \frac{q_{cw}+q_{rw}}{k(T_w-T_m)}\cdot 2y_0 = Nuc_\xi + Nur_\xi \tag{13}$$

$$Nuc_\xi = -2\left.\frac{\partial\theta}{\partial\eta}\right|_{\eta=0}/(1-\theta_m) \tag{14}$$

$$Nur_\xi = \Psi_{rw}/2N_R/(1-\theta_m). \tag{15}$$

Here, Ψ_{rw} denotes the dimensionless radiation heat flux at the wall ($\eta = 0$) and is expressed as

$$\Psi_{rw} = q_{rw}/\sigma T_w^4 = 1-2E_3(\tau_0) - 2\tau_0\int_0^1 \theta^4(\eta')E_2(\tau_0\eta')\,d\eta'. \tag{16}$$

In the present study, we solve equation (6) together with equation (7), utilizing a finite difference scheme [5].

3. APPROXIMATE ONE-DIMENSIONAL METHOD

As seen from equations (13), (14) and (15), the total Nusselt numbers may be readily calculated without solving the two-dimensional energy equation exactly, provided that Nuc_ξ, Ψ_{rw} and θ_m are estimated at each ξ-location by any means.

In the following, we propose an approximate method for evaluating θ_m and Nut_ξ and address its adequacy. First, the governing equation for θ_m can be derived by integrating both sides of equation (6) once with respect to η from 0 to 1

$$\frac{1}{4}\frac{d\theta_m}{d\xi} = Nuc_\xi(1-\theta_m) + \Psi_{rw}/2N_R. \tag{17}$$

The boundary condition for equation (17) is given by

$$\xi = 0 : \theta_m = \theta_0. \tag{18}$$

Equation (17) constitutes a nonlinear ordinary differential equation of the first order with respect to θ_m and can be solved along the flow direction by utilizing a trapezoidal rule, by treating the radiation term as an iterative one at each ξ-location.

It is, however, necessary for the evaluation of Ψ_{rw} to express $\theta(\eta)$ in terms of η and, for this purpose, we assume that $\theta(\eta)$ may be expanded in the form

$$\theta(\eta) = 1 + a_0(1-|2\eta-1|^{2n}), \tag{19}$$

where a_0 and n are unknowns and are determined by the following conditions:

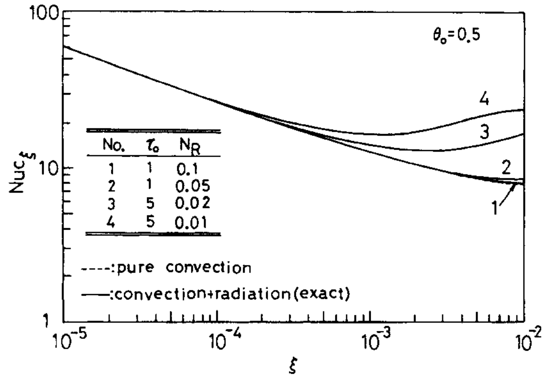

Fig. 1. Variations in the local convective Nusselt number along the flow direction.

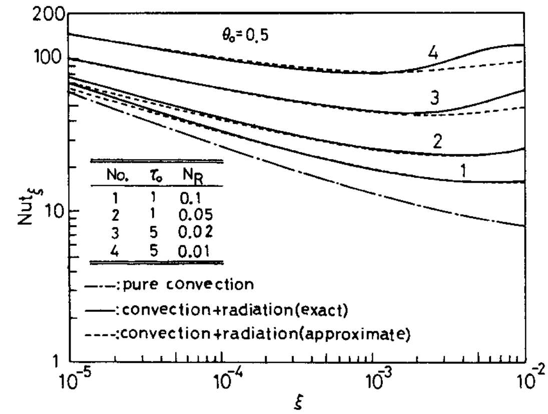

Fig. 2. Variations in the total Nusselt number along the flow direction.

$$Nuc_\xi = -2\frac{\partial\theta}{\partial\eta}\bigg|_{\eta=0}\bigg/(1-\theta_m) = -8a_0 n/1-\theta_m, \quad (20)$$

$$\theta_m = \int_0^1 U\theta\,d\eta = 1+\frac{4a_0 n(n+2)}{(2n+1)(2n+3)}. \quad (21)$$

From these expressions, we obtain

$$\left.\begin{aligned} a_0 &= -Nuc_\xi(1-\theta_m)/8n, \\ n &= [(Nuc_\xi-16)+\sqrt{(16-Nuc_\xi)^2+64Nuc_\xi-192}]/16 \end{aligned}\right\}. \quad (22)$$

Here, we further assume that Nuc_ξ in equations (17) and (22) can be replaced with a correlation for Nusselt numbers of pure convection [6]:

$$Nuc_\xi = 7.541[1+(0.1635\xi^{-1/3}+0.053)^{6.5}]^{1/6.5}. \quad (23)$$

Thus, if a value of θ_m is tentatively given, the temperature profile for Ψ_{rw} is determined from equations (19), (22) and (23) and then the integral appearing in Ψ_{rw} may be estimated by an appropriate means: we utilize a 20th Gaussian quadrature formula. Once θ_m and Ψ_{rw} are determined, Nut_ξ is readily evaluated from its definition, equation (13).

4. RESULTS AND DISCUSSION

For several typical cases, the mixing cup temperatures and total Nusselt numbers are obtained both by the exact two-dimensional and the approximate one-dimensional method. Although variations in the mixing cup temperature along the flow direction are not shown graphically, it was found that the agreement between the exact results and the approximate ones is quite excellent even for the radiation dominant cases corresponding to nos 3 and 4. Figure 1 shows variations in the convective Nusselt number defined by equation (14) along the flow direction. The local convective Nusselt number is well represented by that for pure convection, particularly in the inlet region of a duct, but, for the radiation dominant cases, there occur few derivations from pure convective Nusselt number in the region far from the inlet of a duct. This result suggests that our assumption, as for Nuc_ξ, i.e. equation (23), is fairly reasonable. Figure 2 illustrates the results for the local total Nusselt number. The proposed approximate method predicts well variations in the total Nusselt number along the flow direction: a maximum relative error between the exact and approximate results occurs on no. 4 ($\tau_0 = 5$ and $N_R = 0.01$) and is about 24% at $\xi = 0.01$. Thus, it may be concluded that the proposed approximate method can be utilized for predicting the composite heat transfer characteristics of radiating-gas flow in a black, plane-parallel duct, with an acceptable accuracy.

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